INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
Int. J. Numer. Meth. Fluids 2006; **50**:683–692
Published online 7 September 2005 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fld.1064

A short note on the reattachment length for BFS problem

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SUMMARY

The standard backward-facing step flow problem is solved for steady state laminar case using stream function-vorticity method. The steady state results are obtained as the asymptotic solution of the transient formulation. The primary reattachment length is studied and the discrepancy in the v velocity is reported. A method for determining appropriate locations for comparison is proposed. The energy equation is solved and found to be in good agreement with benchmark results. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: clustered grids; backward step flow; reattachment length; numerical solution; energy solution

1. INTRODUCTION

Fluid flow in a backward-facing step (BFS) geometry is one of the most important benchmark problems used in computational fluid dynamics (CFD). It has an outflow boundary condition, flow separation, reattachment and several recirculation zones. A schematic diagram of the BFS problem is shown in Figure 1. Many authors have studied this benchmark problem and compared their results. However, amongst these studies, there are variations of geometry, types of outflow boundary conditions and numerical techniques to solve the problem.

The earliest significant work is presented by Armaly *et al.* [1]. They have considered the step height and inlet height as 4.9 and 5.2 cm, respectively. The domain length in the downstream direction is considered as 20 cm, sufficient enough for the flow to be fully developed. They made both experimental as well as a two-dimensional (2D) numerical study

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Received 27 May 2005
Revised 29 June 2005
Accepted 30 June 2005

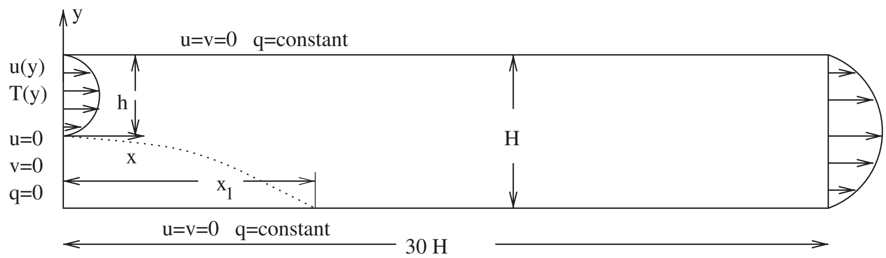

Figure 1. Schematic diagram and boundary conditions in a BFS problem.

of this problem for various Reynolds numbers (*Re*). They found a good agreement between numerical and experimental results up to $Re = 400$. After this *Re*, the numerical results started to deviate from the experimental results. They reported that this discrepancy might be due to the loss of two dimensionality in the experiments. The formation of lower and upper wall vortices and their lengths are described in detail. They concluded that the flow would be laminar up to $Re = 1250$.

Kim and Moin [2] have tested the 3D BFS problem using a fractional-step method. For the intermediate velocity field, suitable boundary conditions are derived and tested. Sohn [3] used FIDAP, the commercial code, to study a few laminar and turbulent flow problems. He reported results of the BFS with and without STU (streamline upwinding).

Gartling [4] studied the problem and made benchmark results for $Re = 800$. The expansion ratio is defined as the ratio of total height of the channel to the step height. Armaly *et al.* [1] used an expansion ratio of 2.061 whereas Gartling [4] and others have used a value of 2. Results presented in Reference [4] included the horizontal velocity component for different downstream locations. It was shown that the streamwise velocity gradients were not zero at the outlet but a constant normal stress condition can be a good approximation as an outflow boundary condition. This work was important from the point of view of a new outflow boundary condition.

The same BFS problem is studied by Dyne and Heinrich [5], and also by Choudhury [6]. In addition to the fluid flow solution, they have solved the energy equation. Dyne and Heinrich [5] investigated the Nusselt (*Nu*) number distribution along the wall. It approaches fully developed value in the downstream direction. Comini *et al.* [7] solved the incompressible 2D flow BFS problem by a stream function-vorticity formulation using finite element method (FEM); they have carried out the computation without upwinding.

Choudhury [6] has reported the reattachment length as 5.8, and showed his u velocity component is in good agreement with Gartling [4] at the downstream location $x = 7$. Barton [8] studied the effect of the inlet channel length before the expansion in the BFS problem and predicted that at low *Re*, the reattachment length is reduced due to the channel presence. Davidson and Nielsen [9] presented the role of the expansion ratio in determining the reattachment length for various *Re*. Bhattacharjee and Loth [10] reported 2D DNS results for the BFS problem as a validation for their code to solve the wall jet flow problem. They reported a primary vortex reattachment length as 5.7. The domain length was 40 times the step height in this study, whereas the benchmark solution of Gartling [4] used 60 times the step height. They have used an extrapolated boundary condition at the exit and reported a reattachment length that is more than a channel height less than that of Gartling [4]. It is worthwhile to point out that Barton [8] also used a shorter domain of 32 times the step height with an extrapolated boundary conditions at the exit; the reported attachment length is close to Gartling [4]. Biswas *et al*. [11] have reported 2D as well as 3D BFS results. They reported the formation of Moffatt eddies as Re approaches zero.

2. PURPOSE OF THIS STUDY

The BFS flow at $Re = 800$ received attention after Kaiktsis *et al*. [12] reported that flow does not have a stable solution above $Re \cong 700$ and the flow undergoes a second bifurcation at $Re = 800$. But Gresho *et al.* [13] concluded that at $Re = 800$ flow is steady and stable solutions are possible. It has been observed that a large number of authors have solved this problem to validate either a new method, new code or new boundary condition. It is found that there are discrepancies in the reattachment length of the primary vortex reported by many authors [1–4, 6, 10, 14], particularly for $Re = 800$ (Table I). This is a reflection of comparing the momentum and energy solutions at different downstream locations with the benchmark values of Gartling [4] and Dyne and Heinrich [5], respectively. This discrepancy can lead to confusion for the new researchers to proceed further with either their method or code. For validation, some authors [7] have used a comparison at relative location instead of the absolute location. The reattachment location at the lower wall for Comini *et al*. [7] is 6.06. They have compared the results with Gartling [4] at $x = 6.96$ instead of $x = 7$. Also it was noticed that although the downstream u velocity matched with the benchmark results of Gartling [4], the v velocity had a large discrepancy with Gartling [4] and Dyne and Heinrich [5] at $x = 7$ (Figure 7(b)); the difference was 68.24% at $y = 0.5$. The present study is aimed at understanding the reasons behind this deviation and the role of reattachment length as a validation parameter in the step flow problem.

Table I. Comparison of primary vortex reattachment length.

Author	x_1/step
Armaly *et al*. (experiment) [1]	14.40
Osswald *et al*. [14]	11.00
Kim and Moin [2]	12.00
Sohn [3]	11.60
Gartling [4]	12.20
Dyne and Heinrich [5]	11.85
Choudhury [6]	11.60
Comini *et al*. [7]	12.12
Barton [8]	12.03
Bhattacharjee and Loth [10]	10.40
Present	11.81

3. NUMERICAL PROCEDURE

The geometry and inlet condition considered here are identical with Gartling [4]. The governing equations for incompressible laminar flow are solved with a stream function-vorticity formulation. At exit the streamwise velocity gradients are assumed to be zero [15]. The transient form of the non-dimensional governing equations are

stream function equation

$$\nabla^2\psi = -\omega \tag{1}$$

vorticity equation

$$\frac{\partial\omega}{\partial t} + \frac{\partial(u\omega)}{\partial x} + \frac{\partial(v\omega)}{\partial y} = \frac{1}{Re}\nabla^2\omega \tag{2}$$

energy equation

$$\frac{\partial\theta}{\partial t} + \frac{\partial(u\theta)}{\partial x} + \frac{\partial(v\theta)}{\partial y} = \frac{1}{RePr}\nabla^2\theta \tag{3}$$

where ψ is the stream function, $u = \partial\psi/\partial y$; $v = -(\partial\psi/\partial x)$; $\omega = (\partial v/\partial x) - (\partial u/\partial y)$; θ is the non-dimensional temperature, $Pr =$ Prandtl number. The solution approaches steady state asymptotically. The computational domains considered here are clustered cartesian grids.

The parabolic equation (2) is solved by an alternate direction implicit (ADI) method [15]. The Poisson equation (1) is solved explicitly by a five-point Gauss–Seidel method. With known velocity values Equation (3) is solved by the ADI method. A constant time step of 0.001 is used for the entire calculation.

4. RESULTS AND DISCUSSION

For the BFS problem, grids are clustered near the high gradients zone (Figure 2). Since grids in both directions are clustered, a systematic grid independence study is carried out for the y-direction as well as the x-direction. In the y-direction different grids systems are tested (Table II) and variation in reattachment length is less than 1% for the third and the fourth

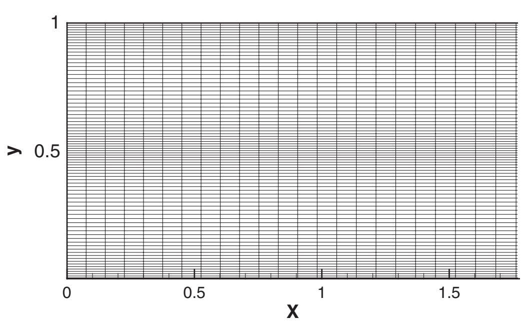

Figure 2. Part of the typical grids used for BFS.

Table II. Grid independence study: $Re = 800$.

Grids in the y-direction	x_1/h
151×41	10.248
151×61	11.168
151×81	11.812
151×101	11.825

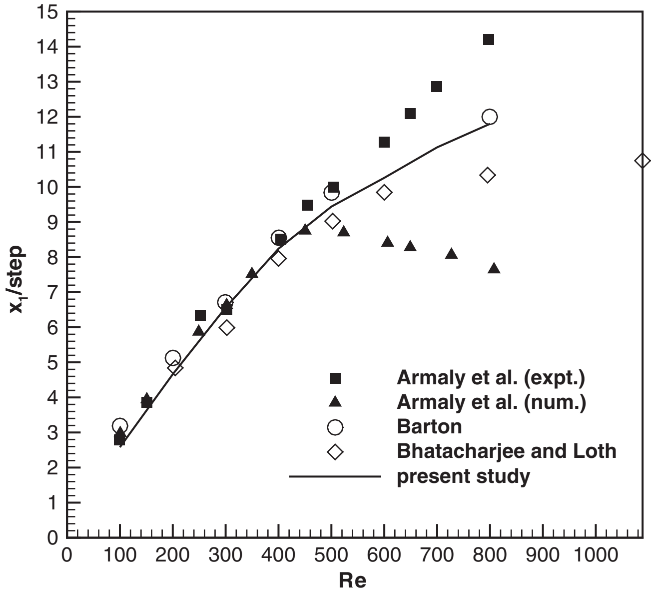

Figure 3. Primary vortex reattachment length for various Re.

grid densities; 81 grids in the y-direction is chosen for the entire study. A similar study is carried out for x-direction with $101 \times 81, 131 \times 81, 151 \times 81$ and 201×81 grid points and it is concluded that 151×81 grid points could be used for all the calculations. For the range of $100 \leqslant Re \leqslant 800$, results are computed and the primary vortex reattachment length is compared (Figure 3) with those of other authors. The reattachment point is determined by interpolation of ω. Most of the researchers have studied the $Re = 800$ case only. Here different line patterns are used to show the present results, while different symbols are used to show the benchmark values. The velocity vectors along with the primary vortex on the bottom wall is shown in (Figure 4(a)) and the top wall vortex is shown in (Figure 4(b)). Also for $Re = 800$, the momentum and energy solutions at various downstream locations are compared with the benchmark results. For the u velocity component (Figure 5(a)), good agreement with benchmark results [4, 5] is demonstrated. Also the energy solution is in good agreement (Figure 5(b)) with Dyne and Heinrich [5]. Table I shows the primary vortex reattachment length reported by few authors.

The upper wall vortex formation begins above $Re = 400$. While Re increases further, the vortex moves in the downstream direction and its length is also increased. The v velocity

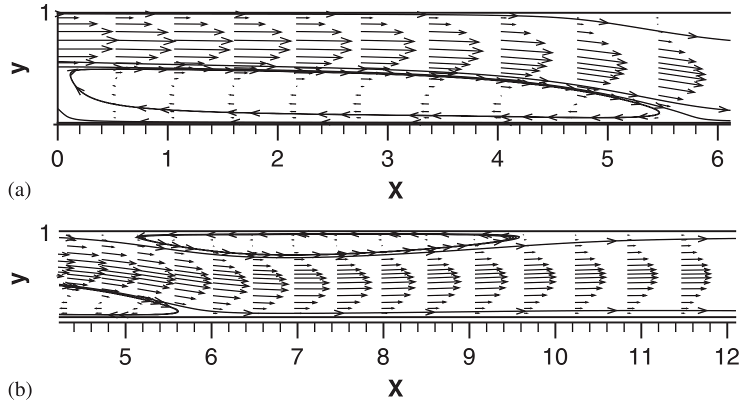

Figure 4. Velocity vector and stream trace plot, $Re = 800$: (a) bottom wall vortex; and (b) upper wall vortex.

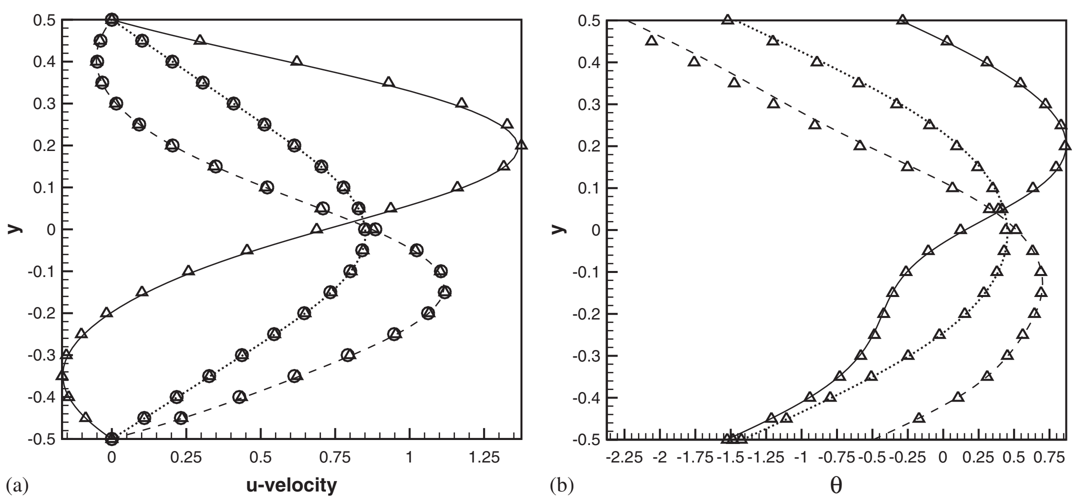

Figure 5. Comparison of numerical solution with benchmark results. O-Gartling, Δ-Dyne and Heinrich and present: — $x = 3$, --- $x = 7$, ... $x = 15$: (a) u velocity; and (b) temperature.

component at $Re = 800$ is compared with the benchmark results (Figure 7). The discrepancy between Gartling [4] and Dyne and Heinrich [5] is to be noted. The v velocity component has a large discrepancy among the present investigation, [4, 5] at downstream locations; in particular, at $x = 7$ it is quite large (Figure 7(b)). Far downstream, the discrepancy is reduced (Figure 6(b)). Thus, it can be concluded that the discrepancy is large just after the primary vortex reattachment. The discrepancy in the v velocity does not significantly affect the energy solution (Figure 5(b)). Barton [8] has concluded that the variation in reattachment length may arise due to the presence of the upstream channel which may be on the order of step height/2, whereas for Bhattacharjee and Loth [10], the reattachment length is lower by 14.74% compared to Gartling [4] which is more than a channel height. Gartling [4] reported the velocity profiles

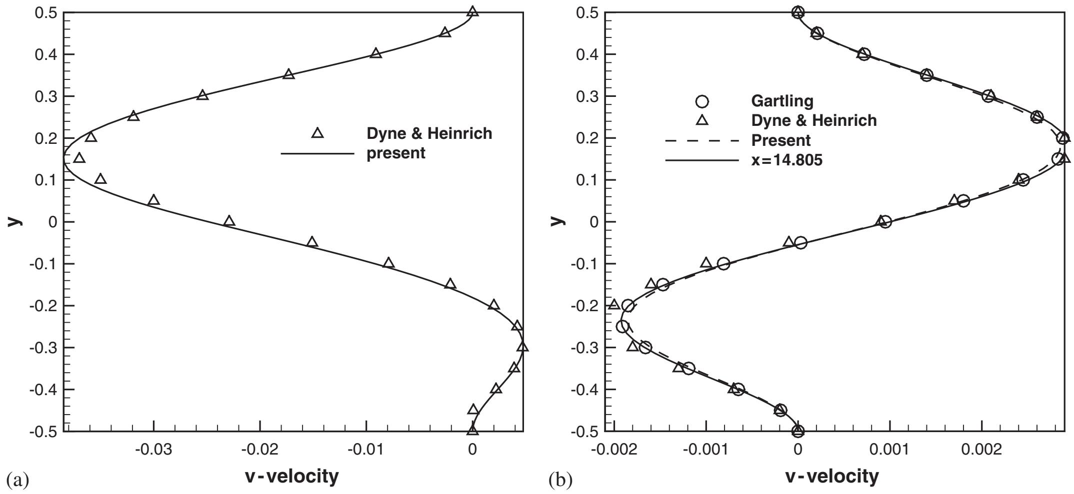

Figure 6. Comparison of v velocity with benchmark results. O-Gartling, Δ-Dyne and Heinrich: (a) $x = 3$; and (b) $x = 15$.

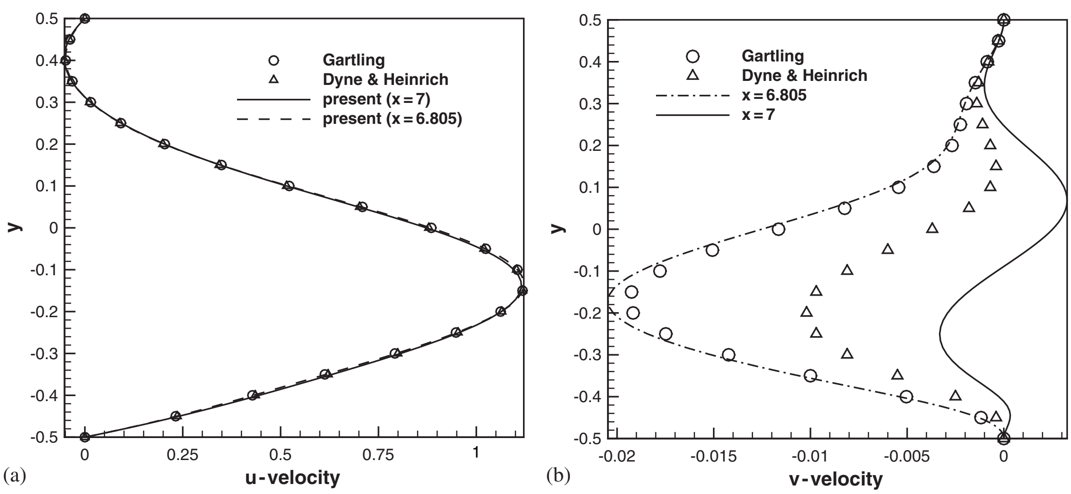

Figure 7. Comparison of velocity at $x = 7$ with benchmark results. O-Gartling, Δ-Dyne and Heinrich: (a) u velocity; and (b) v velocity.

results at $x = 7$ which is $0.9H$ downstream to his reattachment point ($x_1 = 6.1$). Since variations in primary vortex reattachment length are common to most reported solutions (Table I), a method for comparing the velocity profiles in a suitable way has to be identified. In the present solution the reattachment length is 5.905. Instead of proportionate method of Comini *et al.* [7], a shifted location method is used here. Relative to Gartling [4], a value of $0.9H$ is added to the reattachment length, i.e. 5.905, and thus the results are at 6.805. The present results are then compared at this location with the $x = 7$ location results of Gartling [4] (Figure 7(b)) which show good agreement. Both u and v have good agreement in this way (Figure 7(b)).

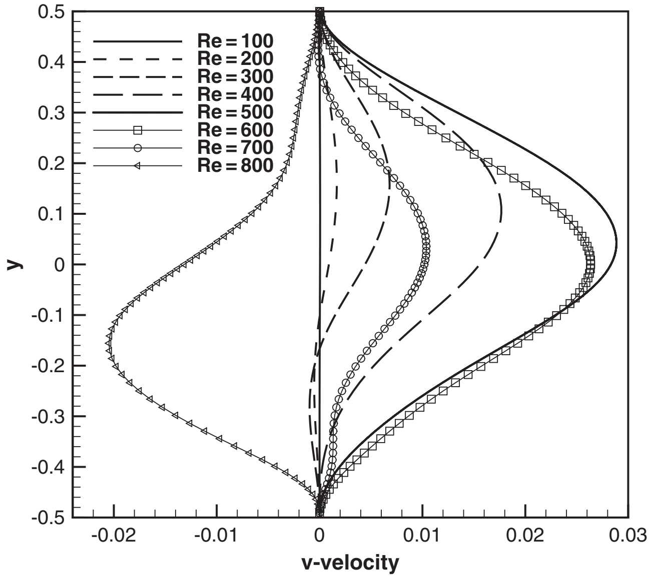

Figure 8. v velocity at $x = 6.805$.

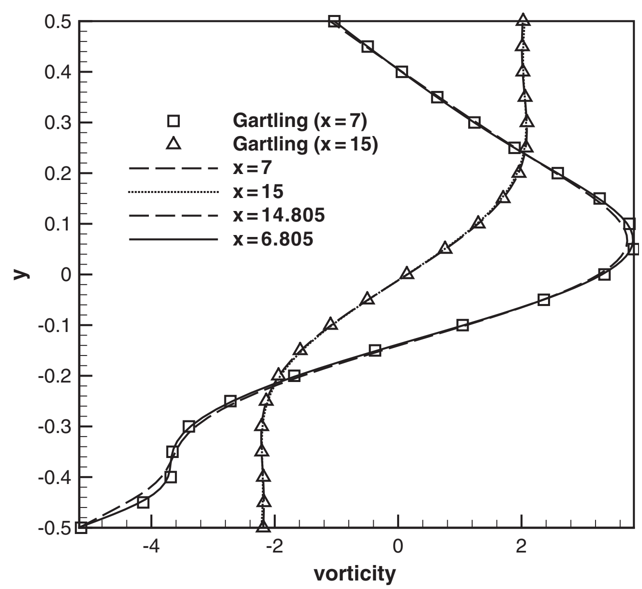

Figure 9. Vorticity at downstream location.

For various Re, the v velocity component is compared at $x = 6.805$ location (Figure 8). At $Re = 100$, the flow is fully developed. $Re = 500$, v increases and with further increase in Re, it becomes negative signifying the appearance of an upper wall vortex. The authors strongly feel that the correct prediction of the v velocity is significant for capturing accurately the primary vortex reattachment length. The discrepancy in the v velocity component is less significant from the heat transfer point of view, because the absolute comparison of θ is in good agreement with Dyne and Heinrich [5] (Figure 5(b)). This may be due to the less convective effect contributed by the v velocity. The vorticity has been compared with Gartling [4] at $x=7$ and 15 and is shown in Figure 9. Unlike the v velocity distribution, for $x=7$, the present computed vorticity is very close to Gartling's [4] results. However, with the proposed shift to $x=6.805$, the lines are overlapping with each other. For $x=15$, it is observed that there is no need of the adjustment.

5. CONCLUSION

The steady state backward-facing step benchmark problem is solved as the asymptotic solution of the time-dependent stream function-vorticity formulation. The discretization has been done by ADI the method with centred space. It is observed that clustering of grids allows use of a lesser number of grid points. The conventional upwinding is not used for the solution procedure. The reattachment length obtained for $Re=800$ is 3.2% less compared to Gartling [4]. Immediately after reattachment of the primary vortex, i.e. at $x=7$, the u velocity predicted is found to match with Gartling [4]. However, the predicted v velocity has a large discrepancy. It is proposed to shift the location for comparison, and thus the predicted v velocity is found to match with Gartling [4]. For locations $x=3$ and 15, it is observed that the computed results compared well with Gartling [4] and no such shift is required. This behaviour is not as pronounced for the vorticity as it is practically matching at $x=7$. The authors feel that the discrepancy in the v velocity leads to the discrepancy in the primary vortex reattachment. It has been found that within the limit of variation of reattachment length in the present study, there is no effect on the temperature distribution for the heat transfer case considered.

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